



Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 4B Further Statistics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

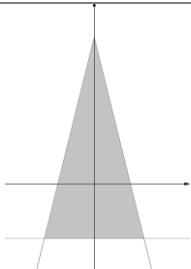
These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \checkmark will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme		Marks	AOs
1(a)	$b = \frac{-338.83}{4.52} [= -74.96...]$		M1	3.3
	$a = \frac{626}{10} - "b" \frac{22.47}{10} [= 231.04.....]$		M1	1.1b
	$t - 40 = "231.04..." + (" - "74.96...") \sqrt{h}$		dM1	3.1a
	$t = 271.04... - 74.96... \sqrt{h}$		A1	1.1b
			(4)	
(b)	Residual = $47 - ("271.04..." - "74.96..." \times \sqrt{9})$		M1	3.4
	= 0.8466...		A1	1.1b
			(2)	
(c)	$RSS = \left[64678 - \frac{626^2}{10} \right] - \frac{(-338.83)^2}{4.52} \left(= 25490.4 - \frac{(-338.83)^2}{4.52} \right)$		M1	1.1b
	= 90.89...[s ²]		A1	1.1b
			(2)	
(d)	Student A's model as the sum of squares of the residuals is lower		B1	2.4
			(1)	
(9 marks)				
Notes:				
(a)	M1	For use of a correct model ie a correct expression for b		
	M1	For use of a correct model ie a correct expression (ft) for a		
	dM1	dep on both previous method marks for proceeding from an equation of the form $v = "a" + "b" w$ to a correct un-simplified model in terms of h and t ft their a and b		
	A1	For a correct model $t = 271.04... - 74.95... \sqrt{h}$ with awrt 271 and awrt 75		
(b)	M1	For a correct method to find the residual. States $h = 9$ and a correct expression to find the residual for their model of the form $t = a + b\sqrt{h}$ or 9 substituted into a correct expression. Must be subtracting the correct way round. Alternatively, may code the data to v and w and attempt $47 - ("231.04..." - "74.96..." \times 3)$		
		awrt 0.85 Allow answers in range awrt 0.84 to awrt 0.86 if working shown following a correct model in (a). If no working is shown then look for awrt 0.847		
(c)	M1	For a correct expression for RSS. 25490.4 may be seen as $\frac{127452}{5}$		
		May also use $RSS = S_{vv}(1 - r^2) = 25490.4 \left(1 - \frac{(-338.83)^2}{4.52 \times 25490.4} \right)$		
	A1	awrt 90.9		
(d)	B1	Explaining a reason for their conclusion that A is a more suitable model provided their positive RSS found in (c) is less than 980. e.g. RSS is smaller so model A Condone references to the model being more accurate oe. Must be a comparison with B or implied so do not accept statements such as “A because it has a small RSS”		

Question		Scheme	Marks	AOs
3(a)		1.6449	B1	3.3
	$30.03 \pm \frac{0.868}{\sqrt{15}} \times "1.6449"$		M1	2.1
	(29.6613..., 30.3986...)		A1	1.1b
			(3)	
(b)		$\chi^2_{14}(0.1)$ CV = 21.064	B1	3.3
	(Reject H_0 if) $\frac{14S^2}{0.868^2} > 21.06$		M1	2.1
	Critical region is $S^2 > 1.133...$		A1	1.1b
			(3)	
(c)		Insufficient evidence that the machine is not working properly as 30.06 is within the confidence interval	M1	2.2b
	and 1.02^2 (1.0404) is not in the CR oe		A1ft	2.4
			(2)	
Total 8				
Notes				
(a)	B1	For realising a normal distribution must be used as a model and finding the correct value 1.6449 or better.		
	M1	For $30.03 \pm \frac{0.868}{\sqrt{15}} \times "z \text{ value}"$. May be implied by a correct CI		
	A1	awrt 29.7 and 30.4 from correct working (B0M1A1 is possible)		
(b)	B1	For realising a chi squared distribution must be used as a model and CV = awrt 21.06		
	M1	Correct method comparing $\frac{14S^2}{0.868^2}$ to $19 < \chi^2_{14} < 24$ (condone equals instead of $>$) Ignore any additional calculations outside of this range.		
	A1	Correct CR allow awrt 1.13 only		
(c)	M1	Dependent on an answer to part (a) or part (b) in order to draw an inference. Drawing a correct inference that there is insufficient evidence to suggest that the machine is not working properly following through one of their CR or CI with one correct comparison. Must mention the <u>machine</u> at least once. If they have either 30.06 outside their CI or 1.02^2 in their CR then allow an inference that the machine is not working properly for this mark following a correct comparison and no contradictory statements relating to that comparison. This mark can still be scored if there is a second comparison which is incorrect.		
	A1ft	Dependent on 30.06 in their confidence interval and 1.02^2 not in the CR. For drawing a correct inference following through their CR or CI with both correct comparisons and no contradictory statements. Must mention the <u>machine</u> at least once. Do not accept a comparison of 30.06 with just one of the limits for the CI.		

Question	Scheme	Marks	AOs
4(a)	$P(G > 12) = \frac{3}{18} \left[= \frac{1}{6} \right]$	B1	1.1b
		(1)	
(b)	$w = 18$	B1	1.1b
	Probability both greater than 12 = $\frac{1}{6} \times \frac{18-12}{18-2}$ oe	M1	1.1b
	$= \frac{1}{16} *$	A1*	1.1b
		(3)	
(c)		M1	3.1a
	x coordinate of Base of triangle = $\pm \left(\frac{k+3}{4} \right)$	M1	1.1b
	Area = $\frac{1}{2} \times 2 \times \left(\frac{k+3}{4} \right) \times (k+3)$ oe	M1	1.1b
	$E(A) = \int_5^{10} \frac{1}{5} \times \frac{1}{4} (k+3)^2 dk$	M1	3.4
	$= \frac{337}{12}$	A1cso	1.1b
		(5)	
(9 marks)			
Notes			
(a)	B1	Allow 0.167 or better isw	
(b)	B1	18	
	M1	Correct calculation shown using their value for w may be implied by use e.g. $\frac{1}{6} \times \frac{3}{8}$	
	A1*	Allow 0.0625 oe	
(c)	M1	Correct triangle identified. Look for a sketch of an isosceles triangle symmetrical about the y -axis with the highest point on the positive y -axis and base vertices below the x -axis. (maybe implied by a horizontal line below the x -axis. Ignore any labelling of vertices if incorrect but may help to indicate positions of vertices. Implied by all three coordinates or calculation to find the area / awrt 28.1	
	M1	For finding either x coordinate of base of triangle $\pm \left(\frac{k+3}{4} \right)$ or implied by $\frac{k+3}{2}$ Do not accept use of a numerical value for k but condone $E(K)$	
	M1	Finding the area of the triangle in terms of k but condone $E(K)$	
	M1	Using the model correctly, for $\int_5^{10} f(k) \times \text{"their area"} dk = \int_5^{10} \frac{1}{5} \times \text{"their area"} dk$ or $E(K) = 7.5$ and uses $E(K^2) = \text{Var}(K) + (E(K))^2$ and uses their Area = $\frac{1}{4}(k^2 + 6k + 9)$	
	A1	awrt 28.1	

Question	Scheme		Marks	AOs
5(a)	$\int ax^{-2} - bx^{-3} dx = -\frac{a}{x} + \frac{b}{2x^2}$		M1	1.1b
	$\left[-\frac{a}{x} + \frac{b}{2x^2} \right]_2^\infty = 0 - \left(-\frac{a}{2} + \frac{b}{8} \right) \qquad \left[= \frac{a}{2} - \frac{b}{8} \right]$ or $\left[-\frac{a}{x} + \frac{b}{2x^2} \right]_2^4 = \left(-\frac{a}{4} + \frac{b}{32} \right) - \left(-\frac{a}{2} + \frac{b}{8} \right) \qquad \left[= \frac{a}{4} - \frac{3b}{32} \right]$ or $\left[-\frac{a}{x} + \frac{b}{2x^2} \right]_4^\infty = 0 - \left(-\frac{a}{4} + \frac{b}{32} \right) \qquad \left[= \frac{a}{4} - \frac{b}{32} \right]$		M1 M1	1.1b 1.1b
	$\frac{a}{2} - \frac{b}{8} = 1$ or $4a - b = 8$ oe $\frac{a}{4} - \frac{3b}{32} = \frac{3}{8}$ or $8a - 3b = 12$ oe $\frac{a}{4} - \frac{b}{32} = \frac{5}{8}$ or $8a - b = 20$ oe		dM1 A1	1.1b 1.1b
	$\therefore a = 3 *$		A1*cs0	2.1
		(6)		
(b)	$b = 4$		B1	1.1b
	$\left[-\frac{3}{x} + \frac{4}{2x^2} \right]_2^m = 0.5$ or e.g. $-3x^{-1} + 2x^{-2} + 1 = 0.5$		M1	1.2
	$m^2 - 6m + 4 = 0$ oe		A1	1.1b
	$(m =) 3 + \sqrt{5}$		A1	2.2a
			(4)	
(10 marks)				
Notes:				
(a)	M1	Attempt to integrate one term correct. Look for the power increasing by 1		
	M1	Integrating both terms and substitute limits the correct way round [any one of $(2, \infty)$ or $(2, 4)$ or $(4, \infty)$] to form one expression where c is a non-zero constant. Alternatively, allow this mark for: Allow this mark for $F(x) = \left(-\frac{a}{x} + \frac{b}{2x^2} \right) - \left(-\frac{a}{2} + \frac{b}{8} \right)$ or one of $F(4)$ or $F(\infty)$		
	M1	Integrating and substitute limits the correct way round [any one of $(2, \infty)$ or $(2, 4)$ or $(4, \infty)$] to form a second expression. Alternatively, for $F(4)$ and $F(\infty)$		
	dM1	Dependent on the 2 nd M. For one of the expressions equal to correct value from 1, $\frac{3}{8}$ or $\frac{5}{8}$		
	A1	Translating a problem in mathematical context into two correct equations with one a term, one b term and one number		
	A1*cs0	Fully correct solution, achieving $a = 3$		

Alt (a)	M1	As in main scheme
	M1	substitutes two of 2, 4 or “ ∞ ” into their integral which must have a constant of integration eg $-\frac{a}{2} + \frac{b}{8} + c (=0)$, $-\frac{a}{4} + \frac{b}{32} + c \left(=\frac{3}{8} \right)$ or $-\frac{a}{\infty} + \frac{b}{\infty} + c (=1)$ oe
	M1	substitutes all three of 2, 4 or “ ∞ ”
	dM1	forms at least two equations involving c eg $-\frac{a}{2} + \frac{b}{8} + c = 0$, $-\frac{a}{4} + \frac{b}{32} + c = \frac{3}{8}$ or $-\frac{a}{\infty} + \frac{b}{\infty} + c = 1$ (may just state $c = 1$)
	A1A1	As in main scheme
(b)	B1	Writing or using $b = 4$ may be seen in (a)
	M1	Equating their integral with b , limits 2 and m substituted and equated to 0.5 Allow their $F(x) = 0.5$. It must be of the form $\alpha x^{-1} + \beta x^{-2} + \gamma = 0.5$ oe May be in terms of eg x instead of m
	A1	A correct 3 term quadratic $= 0$ Terms do not need to be collected on the same side. May be implied by ans. May be in terms of eg x
	A1	$3 + \sqrt{5}$ and any other solutions should be eliminated

Question	Scheme	Marks	AOs
6(a)	The samples are not independent	B1	3.5b
		(1)	
(b)	They should consider the birth weight, gender, or whether or not the lambs are premature. oe	B1 B1	2.4 2.4
		(2)	
(c)	Need the assumption that the underlying distribution of the difference between the weight gains must be normally distributed .	B1	2.4
		(1)	
(d)	Difference -0.9 0.5 0.8 0.2 0.8 0.3 0.1 -0.1	M1	3.1b
	$\bar{w} = 0.2125$ $s^2 = 0.3041...$ ($s = 0.551...$)	M1	1.1b
	Confidence interval : $0.2125 \pm t \times \sqrt{\frac{0.3041}{8}}$	M1	2.1
	$0.2125 \pm 2.998 \times \sqrt{\frac{0.3041}{8}}$	A1ft	1.1b
	$= (-0.37202..., 0.79702...) \text{ oe}$	A1	1.1b
		(5)	
(e)	$H_0 : \mu_w = 0.2$ $H_1 : \mu_w > 0.2$	B1	2.5
	200g = 0.2 kg is in the interval	M1	2.1
	There is no evidence that μ_w is greater than 0.2 oe	A1ft	2.2b
		(3)	
Total 12			
Notes			
(a)	B1	The idea that samples are not independent. Condone other irrelevant comments provided they do not contradict this.	
(b)	B1	For one suitable comment on twins being identical relating to selecting the sample. Condone start weight. Do not accept age / diets of the lambs	
	B1	For a second suitable comment.	
(c)	B1	Need the emboldened words.	
(d)	M1	attempting differences (at least 4 correct) implied by awrt 0.304 or 0.551 but not 0.2125	
	M1	attempt to find \bar{w} and s or s^2 for their differences implied by 0.2125 and either awrt 0.304 or 0.551	
	M1	For using the correct formula their $\bar{w} \pm t \times \sqrt{\frac{\text{their } s^2}{8}}$ where $ t > 2$ all values need to be substituted in.	
	A1ft	for their $\bar{w} \pm \text{awrt } 2.998 \times \sqrt{\frac{\text{their } s^2}{8}}$ all values need to be substituted in	
	A1	dependent on all previous method marks (awrt -0.372, awrt 0.797)	
	SC	If they have carried out a CI for two independent samples allow 3 rd M for using their difference of their means and a pooled variance and A1ft using correct formula with awrt 2.624	
(e)	B1	For both hypotheses correct in terms of μ or μ_w Condone 200	
	M1	For changing 200 g to 0.2 kg (ignore units for this mark) and comparing to their CI	
	A1ft	Independent of hypotheses. Drawing a correct inference following through on their CI provided 0.2 is within their confidence interval, with no contradictory statements. Does not need to be in context. Accept “insufficient evidence to support the (researcher’s) belief”.	

Question	Scheme		Marks	AOs
7(a)	$E(Q) = k \left(\frac{E(X)}{m} + \frac{E(Y)}{n} \right)$			
	$E(Q) = k \left(\frac{mp}{m} + \frac{np}{n} \right)$		M1	3.3
	$2kp = p$ therefore $k = \frac{1}{2} *$		A1*	1.1b
			(2)	
(b)	$E(R) = \frac{amp}{m} + \frac{bnp}{n}$		M1	3.4
	$\frac{amp}{m} + \frac{bnp}{n} = p \therefore a + b = 1 *$		A1*	1.1b
			(2)	
(c)	$\text{Var}(Q) = \frac{mp(1-p)}{m^2} + \frac{np(1-p)}{n^2} \quad \left[= \frac{1}{4} p(1-p) \left(\frac{1}{m} + \frac{1}{n} \right) \right]$		M1	2.1
	$\text{Var}(R) = \frac{a^2 mp(1-p)}{m^2} + \frac{b^2 np(1-p)}{n^2} \quad \left[= p(1-p) \left(\frac{a^2}{m} + \frac{b^2}{n} \right) \right]$		M1	2.1
	$\left(\frac{a^2}{m} + \frac{b^2}{n} \right) < \frac{1}{4} \left(\frac{1}{m} + \frac{1}{n} \right)$		M1	1.1b
	$\left(\frac{a^2}{100} + \frac{(1-a)^2}{200} \right) < \frac{1}{4} \left(\frac{1}{100} + \frac{1}{200} \right)$		M1	1.1b
	$12a^2 - 8a + 1 < 0 \Rightarrow a = ...$		M1	1.1b
	$a = \frac{1}{6}$ or $\frac{1}{2}$		A1	1.1b
	$\frac{1}{6} < a < \frac{1}{2}$		A1ft	2.2a
			(7)	
(11 marks)				
Notes				
(a)	M1	For selecting the correct models for X and Y and subst into $E(Q) = k \left(\frac{E(X)}{m} + \frac{E(Y)}{n} \right)$ Both mp and np must be seen or used as the expected values for X and Y . Allow to be implied by $k \left(E \left(\frac{X}{m} \right) + E \left(\frac{Y}{n} \right) \right) \Rightarrow k(p + p)$ for this mark. Cannot be implied by $k(p + p)$		
	A1*	Cao sets their expression in k and p equal to p before achieving the given answer with no errors.		
(b)	M1	Using the model to find $E(R)$ in terms of a and b . Both mp and np must be seen or used as the expected values for X and Y . Must see $\frac{amp}{m} + \frac{bnp}{n}$ for this mark. Cannot be implied by $ap + bp$		
	A1*	Cao sets their expression in a , b and p equal to p before achieving the given answer with no errors.		

(c)	M1	For a correct attempt at $\text{Var}(Q)$ with at least two of m, n and 2 being squared on the denominator. May be implied if they cancel by m or n
	M1	<p>For a correct attempt at $\text{Var}(R)$ in terms of a and b with at least one of a and b being squared and at least one of m and n being squared. May be implied if they cancel by m or n</p> <p>Expression may be in a only</p> <p>e.g. $\text{Var}\left(\frac{aX}{m} + \frac{bY}{n}\right) = \text{Var}\left(\frac{aX}{m} + \frac{(1-a)Y}{n}\right) = \frac{a^2 mp(1-p)}{m^2} + \frac{(1-a)^2 np(1-p)}{n^2}$</p> <p>Note attempting $\text{Var}\left(\frac{aX}{m} + \frac{bY}{n}\right) = \text{Var}\left(\frac{aX}{m} + \frac{(1-a)Y}{n}\right) = \text{Var}\left(\frac{aX}{m} + \frac{Y}{n} - \frac{aY}{n}\right)$ is M0</p>
	M1	using their $\text{Var}(R) < \text{their Var}(Q)$ condone = instead of $<$ (there must be a term in a^2 in their equation or inequality)
	M1	substituting their $b = 1 - a$ may be scored earlier
	M1	forming and solving correctly a 3 term quadratic in a . condone = instead of $<$
	A1	correct values
	A1ft	Dep. on all previous M marks and for selecting the right range using their values of a which must be between 0 and 1

Question		Scheme	Marks	AOs
8(a)		Let $T = S_1 + S_2 + S_3$ then $E(T) = 1500$	M1	3.3
		$\text{Var}(T) = 75$	M1	2.1
		$P(1490 < T < 1530) = 0.8756\dots$	A1	1.1b
			(3)	
(b)		Let $W = \pm(L - 2S - 30)$ then $E(W) = \pm(1020 - 2 \times 500 - 30)$ or Let $X = \pm(L - 2S)$ then $E(X) = \pm(1020 - 2 \times 500)$	M1	3.3
		$E(W) = -10$ (or 10) or $E(X) = 20$ (or -20)	A1	1.1b
		$\text{Var}(\dots) = 20^2 + 4 \times 5^2$	M1	2.1
		$\text{Var}(\dots) = 500$	A1	1.1b
		$P(W > 0)$ or $P(X > 30)$ (or $P(W < 0)$ or $P(X < 30)$)	M1	2.1
		$= 0.3273\dots$	A1	1.1b
			(6)	
(9 marks)				
Notes				
(a)	M1	Selecting and using the appropriate model and attempting 3×500		
	M1	For realising the need to use $\text{Var}(S) + \text{Var}(S) + \text{Var}(S) = 3 \times 5^2$		
	A1	awrt 0.876		
(b)	M1	Selecting and using the appropriate model $\pm(L - 2S - 30)$ or $\pm(L - 2S)$ in an attempt to find the expected value		
	A1	-10 or 20 (or 10 or -20)		
	M1	For realising they need to use $\text{Var}(L) + 4\text{Var}(S) = 20^2 + 4 \times 5^2$		
	A1	500 only		
	M1	dependent on using an appropriate model and realising that $P(W > 0)$ (or $P(W < 0)$) or $P(X > 30)$ (or $P(X < 30)$) is required. May be implied by awrt 0.327 Using standardisation look for e.g. $P\left(Z > \frac{0 - "-10"}{\sqrt{500}}\right)$ or $P\left(Z > \frac{30 - "-20"}{\sqrt{500}}\right)$ ($= P(Z > 0.4472\dots)$)		
	A1	awrt 0.327		

